Integration of quantum theory and relativity theory via a new perspective on relativistic transformation

Young-Sea Huang*

Department of Physics, Soochow University, Shih-Lin, Taipei 111, Taiwan

We argue that special relativity, instead of quantum theory, should be radically reformulated to resolve inconsistencies between those two theories. The idea for a new theory of relativity consistent with quantum theory is outlined. A new relativistic transformation recently-proposed renders physical laws form-invariant via transformation of physical quantities, instead of space-time coordinates. Maxwell's equations of electrodynamics are rendered form-invariant among inertial frames by this new relativistic transformation. The space-time concept underlying this new relativistic transformation is Newtonian absolute space and absolute time as presumed in quantum theory. This new perspective on relativistic transformation provides an insight into the very meaning of the principle of relativity. The principle of relativity means that the same physical laws hold in all inertial frames, rather than their mathematical formulas are Lorentz-covariant under the Lorentz transformation of space-time coordinates. With this new perspective on relativistic transformation, quantum theory becomes compatible with the principle of relativity.

PACS numbers: Quantum mechanics, 03.65.-w; Special relativity, 03.30.+p; Quantum entanglement, 03.65.Ud; Philosophy of science, 01.70.+w, *43.10.Mq
Keywords: conflict between quantum theory and special relativity, quantum theory, special relativity, the principle of relativity, Lorentz covariance, non-locality, indeterminacy, Bell inequality, entanglement, realism, completeness

I. CONFLICT BETWEEN RELATIVITY THEORY AND QUANTUM THEORY

Einstein's relativity theory and quantum theory are the two revolutionary theories evolving from classical physics theory. Nowadays, they are the two pillars of modern physics, for example, relativistic quantum mechanics and quantum electrodynamics are constructed upon them. In spite of Einstein's relativity theory and quantum theory being, respectively, well verified experimentally, debates on the foundational conflict between these two theories seem endless [1–15]. Einstein's relativity theory overthrows Newtonian absolute space and absolute time as presumed in classical theory, whereas quantum theory is formulated on that conventional notion of space and time. On how the physical world could be described, relativity theory adopts some basic notions of classical theory: there is no instantaneous action at a distance, and a physical theory should be able to predict definite values for all physical quantities pertaining to a system in the physical world. Quantum theory, in contrast, provides some perspectives on the physical world that are radically different from basic notions of classical theory: non-locality, quantum of action, indeterminacy, and probabilistic interpretation of waves functions.

Are relativity theory and quantum theory really incompatible? Debates on this subject are related to such issues as non-locality, realism and completeness, raised by the well-known EPR article of Einstein, Podolsky and Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?* [16]. With the belief that for each element of physical reality a corresponding physical quantity should have a definite value prior to measurement (realism), and their necessary condition for completeness of a physical theory as well as criterion for an element of reality [17], applying the classical locality [18], Einstein *et al* showed that from the quantum-mechanical wave function of two entangled particles, exact values of the position Q and the momentum P of one particle can be inferred from respective measurements on the position q and the momentum p of the other particle carried out at a distantly separated place (far from the particles' interaction). Then, by what they term *reasonable definition of reality*, they argued that a particle possesses simultaneous elements of reality for the non-commuting observables of position Q and momentum P. Yet, according to Heisenberg's uncertainty principle, simultaneous values of position and momentum of a particle can not be exactly determined by the wave function. Supposing quantum theory is correct, either quantum-mechanical description of physical phenomena is not complete, or these two non-commuting observables of position Q and momentum P can not have simultaneous elements of reality. Thus, there exist elements of physical reality whose corresponding physical quantities can not be predicted, with certainty, by quantum theory. Einstein

*Electronic address: yshuang@mail.scu.edu.tw

et al concluded that quantum theory must at best be incomplete. An anticipated complete theory, with additional variables not known yet, should be able to predict simultaneous values of position and momentum of a particle. Bohr rebutted the EPR argument, and contended that quantum-mechanical description of physical phenomena fulfills, within its scope, all rational demands of completeness [19]. The notions of classical locality and realism advocated in the EPR argument are incompatible with Heisenberg's uncertainty principle and the probabilistic interpretation of quantum theory. Initially, those issues raised in the EPR argument seem to have been considered merely as subjects for philosophical debate. Times change.

To present the EPR argument in a simplified form, Bohm and Aharonov considered a special example − a system of two spin-1/2 particles is created in the singlet spin state and these two particles then fly off in opposite directions [20]. Using this example, Bell studied the correlation of the spin states of the two particles of which directions of spin measured by two spatially-separated detectors were allowed to be oriented arbitrarily, instead of orienting them along the same direction [21]. Bell discovered that the correlation of the spins of the two particles predicted by any theory, relativity theory included, that requires the classical locality, must obey the so-called Bell Inequality. In contrast, the correlation of the spins of the two particles predicted by quantum theory violates the Bell Inequality. Bell's discovery made those seemingly philosophical issues raised by the EPR argument become testable by experiments. Many experiments on generalized Bell's Inequalities, for example, the CHSH inequality[22], were performed, and their results have so far been in agreement with the predictions of quantum theory [23–27]. Those experimental tests indicate not merely that relativity theory and quantum theory are incompatible, but that relativity theory is probably not valid.

Either options: that relativity theory and quantum theory are incompatible or that relativity theory is invalid, entails a catastrophe to the foundation of modern physics. The validity of relativistic quantum theory and relativistic electrodynamics are called into question, since their foundations are insecure. Many suggestions to resolve the conflict between quantum theory and relativity theory have been proposed, but none of them are totally convincing so far. It is still unclear how to get a consistent description of the physical world out of these two mutually-incompatible theories [7]. Though most physicists seem unwilling to give up relativity theory, we think that relativity theory should be radically reformulated to reconcile with quantum theory, since experimental tests of the Bell Inequality so far indicate that relativity theory is probably wrong.

II. A NEW PERSPECTIVE ON RELATIVISTIC TRANSFORMATION

One major conflict between Einstein's relativity theory and quantum theory is rooted in their mutually contradictory notions of space and time [1–6]. According to Einstein's relativity theory, simultaneity of two space-like events is no longer absolute, that is, the time order of the events depends on reference frames used to describe the events. Contrarily, quantum theory is formulated on Newtonian absolute space and absolute time. Simultaneity is absolute in quantum theory; without it, Heisenberg's uncertainty principle and the probabilistic interpretation of quantum theory become meaningless. To be consistent with quantum theory, an anticipated new theory of relativity should be necessarily formulated on the same concept of space and time as is presumed in quantum theory.

A new relativistic transformation was recently formulated on the two postulates, the principle of relativity and the constancy of speed of light, the same as postulated in Einstein's special relativity [28]. Yet, this new relativistic transformation presumes Newtonian absolute space and absolute time, whereas Einstein's special relativity does not. Since most physicists are unfamiliar with this new relativistic transformation, we briefly recapitulate it.

Consider a particle moving in an inertial reference frame X. With respect to the frame, during an infinitesimal time interval $\mathrm{d}t > 0$, the particle will move with a spatial displacement $\mathrm{d}\mathbf{x} = \mathbf{v}\,\mathrm{d}t$, where $\mathbf{v}$ is the instantaneous velocity of the particle. Since the speed of light c is an invariant as postulated, we define $\mathrm{d}x^0 \equiv c\,\mathrm{d}t$. Then, one can use the four-vector of infinitesimal displacement $\mathrm{d}x^\alpha \equiv (\mathrm{d}x^0, \mathrm{d}\mathbf{x})$ to characterize the state of motion of the particle. It should be noted that according to Heisenberg's uncertainty principle it is impossible to definitely know the position $\mathbf{x}$ of a particle when its velocity (hence momentum) is exactly determined, that is, when its state of motion $\mathrm{d}x^\alpha$ is exactly specified. Any attempt to determine the position of the particle $\mathbf{x}$ will disturb its state of motion $\mathrm{d}x^\alpha$. Both the state of motion $\mathrm{d}x^\alpha$ and the position $\mathbf{x}$ of a particle can not be exactly determined at the same time. Therefore, it is impossible to have a relativistic transformation for both the state of motion $\mathrm{d}x^\alpha$ and the position $\mathbf{x}$ of a particle.

Consider another inertial reference frame X' moving with a constant velocity $\mathbf{V}$ with respect to the inertial reference frame X. At an arbitrary instant of time, a particle has the state of motion $\mathrm{d}x^\alpha$ with respect to the frame X, and has the corresponding state of motion $\mathrm{d}x'^\mu$ with respect to the frame X'. Suppose that there exists a universal transformation on the state of motion, and the transformation is linear. That is,

$$\mathrm{d}x^\alpha = L^\alpha_\mu(X, X')\,\mathrm{d}x'^\mu \quad (\alpha, \mu = 0, 1, 2, 3). \tag{1}$$

The transformation is assumed to be linear, since particles in uniformly rectilinear motion with respect to an inertial

frame must also be in uniformly rectilinear motion with respect to all other inertial frames. The coefficients $L^{\alpha}_{\mu}(X,X')$ depend on the relationships between the coordinate system of the frame X and the coordinate system of the frame X', such as the orientation of coordinate systems and the relative velocity $\mathbf{V}$ between the reference frames, but not at all on the motion of the particle.

For the case that the frame X' moves with the speed V along the positive X^1-axis with respect to the frame X, applying the two postulates, the principle of relativity and the constancy of speed of light, we obtain the relativistic transformation for $\mathrm{d}x^{\alpha}$

$$\begin{cases} \mathrm{d}x^0 = & \gamma(\mathrm{d}x'^0 + \beta\,\mathrm{d}x'^1) \\ \mathrm{d}x^1 = & \gamma(\beta\,\mathrm{d}x'^0 + \mathrm{d}x'^1) \\ \mathrm{d}x^2 = & \mathrm{d}x'^2 \\ \mathrm{d}x^3 = & \mathrm{d}x'^3 \ , \end{cases} \tag{2}$$

where $\beta = V/c$ and $\gamma = 1/\sqrt{1-\beta^2}$. We name this new relativistic transformation the differential Lorentz transformation. This new relativistic transformation is a transformation in the physical space, i.e., the space of physical quantities $\mathrm{d}x^{\alpha}$.

The relativistic four-momentum of a free particle of rest mass m moving with velocity $\mathbf{v}$ with respect to a frame is defined as $p^{\alpha} = m\,c\,\mathrm{d}x^{\alpha}/\mathrm{d}\tau$. Here, $\mathrm{d}\tau \equiv (\mathrm{d}x^{\alpha}\,\mathrm{d}x_{\alpha})^{1/2} = \mathrm{d}x^0/\gamma_v$ is an invariant under the differential Lorentz transformation, where $\gamma_v \equiv 1/\sqrt{1-(v/c)^2}$. From p^{α} for the free particle, we have the well-known relativistic energy $E = p^0\,c = \gamma_v\,mc^2$ and relativistic momentum $\mathbf{p} = \gamma_v\,m\,\mathbf{v}$. Also, the relativistic transformation of the four-momentum between frames X and X' is

$$\begin{cases} \mathrm{d}p^0 = & \gamma(\mathrm{d}p'^0 + \beta\,\mathrm{d}p'^1) \\ \mathrm{d}p^1 = & \gamma(\beta\,\mathrm{d}p'^0 + \mathrm{d}p'^1) \\ \mathrm{d}p^2 = & \mathrm{d}p'^2 \\ \mathrm{d}p^3 = & \mathrm{d}p'^3 \ . \end{cases} \tag{3}$$

Equivalently, the differential Lorentz transformation (DLT) can be considered as a transformation in the physical space of energy-momentum.

According to Einstein's special relativity, the currently-accepted relativistic transformation is the Lorentz transformation (LT) of space-time coordinates

$$\begin{cases} x^0 = & \gamma(x'^0 + \beta\,x'^1) \\ x^1 = & \gamma(\beta\,x'^0 + x'^1) \\ x^2 = & x'^2 \\ x^3 = & x'^3 \ . \end{cases} \tag{4}$$

The DLT of displacements Eq. (2) is usually thought as just a derivative of the LT of space-time coordinates Eq. (4). Contrarily, they are not equivalent. The infinitesimal quantities $\mathrm{d}x^{\alpha}$ in the DLT are not the differential of the space-time coordinates x^{α} in the LT [29]. To misconceive it that way, as mentioned, contradicts the basics of quantum theory.

To explicate that the new DLT and the usual LT are not equivalent, consider light waves propagating in a medium which moves at "superluminal" speeds opposite to the propagation direction of waves [30]. Referring to Fig. 1, relative to the medium rest frame X', light waves propagate in the positive x-axial direction with speed $v' = \omega'/k'$, where ω' is the frequency of the waves ($\omega' > 0$), and k' is the wave vector ($k' > 0$). The frame X' moves with velocity V in the negative x-axial direction with respect to the frame X, and $V > v'$. According to Einstein's relativity theory, by using the invariance of the phase of waves, $\omega\,t - \mathbf{k}\cdot\mathbf{r} = \omega'\,t' - \mathbf{k}'\cdot\mathbf{r}'$, together with the LT Eq. (4), the relativistic transformation of the 4-vector $(\omega/c, \mathbf{k})$ of waves is

$$\begin{cases} \frac{\omega}{c} \ = & \gamma(\frac{\omega'}{c} - \beta\,k'_x) \\ k_x = & \gamma(k'_x - \beta\,\frac{\omega'}{c}) \\ k_y = & k'_y \\ k_z = & k'_z \ . \end{cases} \tag{5}$$

By the transformation of 4-vector $(\omega/c, \mathbf{k})$ of waves Eq. (5), one has $\omega = \gamma\,(1 - V/v')\,\omega' < 0$ and $k = \gamma\,(1 - V v'/c^2)\,k' > 0$, for the light waves propagating with respect to the frame X . Relative to the frame X, the light waves propagate in the positive x-axial direction, but with *negative* frequency. However, physical waves, like light waves, in the physical world can not oscillate with negative frequencies. This anomaly is resolved by this new relativistic transformation [30]. Therefore, the DLT is not just a derivative of the LT of space-time coordinates, as is usually thought.

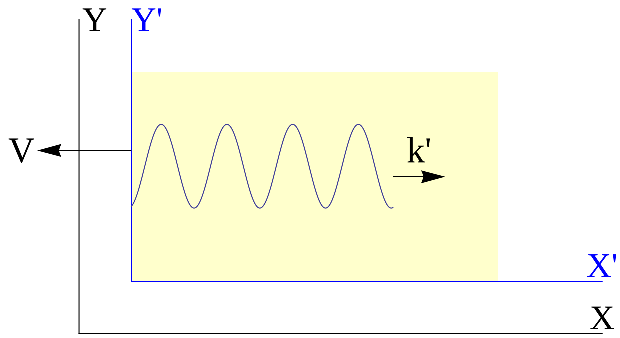

FIG. 1: The medium rest frame X' moves with velocity V in the negative x-axial direction with respect to the frame X. With respect to the frame X', light waves propagate with the wave vector $\mathbf{k}'$ in the positive x-axial direction.

Furthermore, with this new perspective on relativistic transformation, Maxwell's equations of electrodynamics were shown to be form-invariant among inertial frames, via transformation in the physical space, rather than the space-time space [31]. Maxwell's equations of electrodynamics expressed in manifestly covariant form are

$$\partial_\alpha \, \tilde{G}^{\alpha\beta}(\mathbf{r},t) = \tilde{J}^{\beta}(\mathbf{r},t), \tag{6}$$

$$\partial_\alpha \, \tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{r},t) = 0, \tag{7}$$

where $\partial_\alpha \equiv (\partial/\partial(ct), \nabla)$. Here, for mathematical simplicity, we extend real-valued electromagnetic fields to corresponding complex-valued electromagnetic fields. The tilde sign on the top of a symbol is used to indicate the value of that symbol is complex. The complex-valued electromagnetic field tensor $\tilde{G}^{\alpha\beta}(\mathbf{r},t)$ is defined as

$$\begin{pmatrix} 0 & -c\tilde{D}_x(\mathbf{r},t) & -c\tilde{D}_y(\mathbf{r},t) & -c\tilde{D}_z(\mathbf{r},t) \\ c\tilde{D}_x(\mathbf{r},t) & 0 & -\tilde{H}_z(\mathbf{r},t) & \tilde{H}_y(\mathbf{r},t) \\ c\tilde{D}_y(\mathbf{r},t) & \tilde{H}_z(\mathbf{r},t) & 0 & -\tilde{H}_x(\mathbf{r},t) \\ c\tilde{D}_z(\mathbf{r},t) & -\tilde{H}_y(\mathbf{r},t) & \tilde{H}_x(\mathbf{r},t) & 0 \end{pmatrix}, \tag{8}$$

and $\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{r},t)$ is defined as

$$\begin{pmatrix} 0 & -c\tilde{B}_x(\mathbf{r},t) & -c\tilde{B}_y(\mathbf{r},t) & -c\tilde{B}_z(\mathbf{r},t) \\ c\tilde{B}_x(\mathbf{r},t) & 0 & \tilde{E}_z(\mathbf{r},t) & -\tilde{E}_y(\mathbf{r},t) \\ c\tilde{B}_y(\mathbf{r},t) & -\tilde{E}_z(\mathbf{r},t) & 0 & \tilde{E}_x(\mathbf{r},t) \\ c\tilde{B}_z(\mathbf{r},t) & \tilde{E}_y(\mathbf{r},t) & -\tilde{E}_x(\mathbf{r},t) & 0 \end{pmatrix}. \tag{9}$$

Referring to Fig. 2, using plane wave decomposition,

$$\tilde{G}^{\alpha\beta}(\mathbf{r},t) = \int_{-\infty}^{\infty} \tilde{G}^{\alpha\beta}(\mathbf{k}) \, e^{i\,(\mathbf{k}\cdot\mathbf{r} - \omega(\mathbf{k})\,t)} \, \mathrm{d}^3\mathbf{k}, \tag{10}$$

and

$$\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{r},t) = \int_{-\infty}^{\infty} \tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{k}) \, e^{i\,(\mathbf{k}\cdot\mathbf{r} - \omega(\mathbf{k})\,t)} \, \mathrm{d}^3\mathbf{k}, \tag{11}$$

from Eqs. (6) and (7) , we have Maxwell's equations in $\mathbf{k}$-space,

$$\tilde{G}^{\alpha\beta}(\mathbf{k}) k_\beta = -i \, \tilde{J}^{\alpha}(\mathbf{k}), \tag{12}$$

$$\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{k}) k_\beta = 0. \tag{13}$$

Here, $k^\alpha = (\omega/c, \mathbf{k})$, $\tilde{G}^{\alpha\beta}(\mathbf{k})$ is

$$\begin{pmatrix} 0 & -c\tilde{D}_x(\mathbf{k}) & -c\tilde{D}_y(\mathbf{k}) & -c\tilde{D}_z(\mathbf{k}) \\ c\tilde{D}_x(\mathbf{k}) & 0 & -\tilde{H}_z(\mathbf{k}) & \tilde{H}_y(\mathbf{k}) \\ c\tilde{D}_y(\mathbf{k}) & \tilde{H}_z(\mathbf{k}) & 0 & -\tilde{H}_x(\mathbf{k}) \\ c\tilde{D}_z(\mathbf{k}) & -\tilde{H}_y(\mathbf{k}) & \tilde{H}_x(\mathbf{k}) & 0 \end{pmatrix}, \tag{14}$$

and $\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{k})$ is

$$\begin{pmatrix} 0 & -c\tilde{B}_x(\mathbf{k}) & -c\tilde{B}_y(\mathbf{k}) & -c\tilde{B}_z(\mathbf{k}) \\ c\tilde{B}_x(\mathbf{k}) & 0 & \tilde{E}_z(\mathbf{k}) & -\tilde{E}_y(\mathbf{k}) \\ c\tilde{B}_y(\mathbf{k}) & -\tilde{E}_z(\mathbf{k}) & 0 & \tilde{E}_x(\mathbf{k}) \\ c\tilde{B}_z(\mathbf{k}) & \tilde{E}_y(\mathbf{k}) & -\tilde{E}_x(\mathbf{k}) & 0 \end{pmatrix}. \tag{15}$$

$$\begin{tikzcd}[column sep=huge] \text{Maxwell's equations viewed in frame X} \arrow[r, leftrightarrow, dashed, "\textbf{plane wave decomposition}"] & \text{k-space Maxwell's equations viewed in frame X} \arrow[d, leftrightarrow, dashed, "\textbf{Relativistic Transformation in k-space}"] \\ \text{Maxwell's equations viewed in frame } X' \arrow[r, leftrightarrow, dashed, "\textbf{plane wave decomposition}"'] & \text{k-space Maxwell's equations viewed in frame } X' \end{tikzcd}$$

FIG. 2: Maxwell's equations are made form-invariant via the new scheme of relativistic transformation, i.e., transformation in the **k**-space, rather than the space-time space.

If Maxwell's equations Eqs. (6) and (7) are the same in all inertial frames, then Maxwell's equations in k-space Eqs. (12) and (13) are the same in all inertial frames. Consequently, since k^α is a covariant vector, $\tilde{G}^{\alpha\beta}(\mathbf{k})$ and $\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{k})$ are expected to be covariant in k-space, that is,

$$\tilde{G}'^{\alpha\beta}(\mathbf{k}') = L^\alpha_\mu\, L^\beta_\nu\, \tilde{G}^{\mu\nu}(\mathbf{k}), \tag{16}$$

$$\tilde{\mathcal{F}}'^{\alpha\beta}(\mathbf{k}') = L^\alpha_\mu\, L^\beta_\nu\, \tilde{\mathcal{F}}^{\mu\nu}(\mathbf{k}), \tag{17}$$

where L^α_β represents the appropriate relativistic transformation. Then, from the transformed tensors in k-space $\tilde{G}'^{\alpha\beta}(\mathbf{k}')$ and $\tilde{\mathcal{F}}'^{\alpha\beta}(\mathbf{k}')$ with respect to the frame X', we apply plane wave decomposition to construct the transformed electrical and magnetic fields $\tilde{G}'^{\alpha\beta}(\mathbf{r}',t')$ and $\tilde{\mathcal{F}}'^{\alpha\beta}(\mathbf{r}',t')$ with respect to the frame X'. Finally, the transformed electrical and magnetic fields are shown to satisfy Maxwell's equations; that is,

$$\partial'_\alpha\, \tilde{G}'^{\alpha\beta}(\mathbf{r}',t') = \tilde{J}'^{\beta}(\mathbf{r}',t'), \tag{18}$$

$$\partial'_\alpha\, \tilde{\mathcal{F}}'^{\alpha\beta}(\mathbf{r}',t') = 0. \tag{19}$$

Maxwell's equations of electrodynamics are form-invariant among inertial frames under this new relativistic transformation. This new relativistic transformation of electrodynamics fulfills the principle of relativity.

Furthermore, we pointed out that Maxwell's equations transformed by the LT does not truly fulfill the principle of relativity [32]. According to the current relativity theory, Maxwell's equations are Lorentz covariant under the LT by assuming that electromagnetic field tensors, $\tilde{G}^{\mu\nu}(\mathbf{r},t)$ and $\tilde{\mathcal{F}}^{\mu\nu}(\mathbf{r},t)$, are Lorentz covariant, that is,

$$\underline{\tilde{G}}'^{\alpha\beta}(\mathbf{r}',t') = L^\alpha_\mu\, L^\beta_\nu\, \tilde{G}^{\mu\nu}(\mathbf{r},t), \tag{20}$$

$$\underline{\tilde{\mathcal{F}}}'^{\alpha\beta}(\mathbf{r}',t') = L^\alpha_\mu\, L^\beta_\nu\, \tilde{\mathcal{F}}^{\mu\nu}(\mathbf{r},t), \tag{21}$$

under the LT. It should be emphasized that the space-time coordinates in Eqs. (20) and (21) are also under the same LT. Thus, by the LT, Maxwell's equations with respect to the frame X' are

$$\partial'_\alpha\, \underline{\tilde{G}}'^{\alpha\beta}(\mathbf{r}',t') = \underline{\tilde{J}}'^{\beta}(\mathbf{r}',t'), \tag{22}$$

$$\partial'_\alpha\, \underline{\tilde{\mathcal{F}}}'^{\alpha\beta}(\mathbf{r}',t') = 0. \tag{23}$$

Originally, the time t in the fields $\tilde{G}^{\alpha\beta}(\mathbf{r},t)$ and $\tilde{\mathcal{F}}^{\alpha\beta}(\mathbf{r},t)$ is at the same time with respect to the frame X; whereas, the time t' in the transformed fields $\underline{\tilde{G}}'^{\alpha\beta}(\mathbf{r}',t')$ and $\underline{\tilde{\mathcal{F}}}'^{\alpha\beta}(\mathbf{r}',t')$ are at different times with respect to the frame

X', due to the LT of space-time coordinates. This non-simultaneity due to the LT of space-time coordinates poses serious problems of interpreting those fields defined over times extending from past to future. In contrast, by the new relativistic transformation, the time t' in the transformed Maxwell's equations Eqs. (18) and (19) refers to an instant of time with respect to the frame X'.

It was pointed out that the requirement of Lorentz covariance of the laws of physics is not sufficient for the laws to fulfill the principle of relativity [33]. Besides such covariance requirement, all quantities in the covariant equations must be defined and interpreted physically in the same way in all inertial frames. With respect to the frame X', the transformed Maxwell's equations, Eqs. (22) and (23), are not defined and interpreted physically in the same way as Maxwell's equations, Eqs. (6) and (7), with respect to the frame X originally. The LT renders Maxwell's equations Lorentz covariant only in a superficially consistent manner. Contrary to the current viewpoint, the usual way of rendering Maxwell's equations of electrodynamics form-invariant by the LT does not truly fulfill the principle of relativity.

III. THE PRINCIPLE OF RELATIVITY AND QUANTUM THEORY

Can quantum theory be consistent with the principle of relativity? According to the current notion of the principle of relativity in Einstein's relativity theory, mathematical formulas of physical laws must be Lorentz covariant under the LT of space-time coordinates. Then, physical laws are required to be expressed as mathematical formulas of space and time coordinates in order to see whether or not their mathematical formulas satisfy this Lorentz covariance criterion of the principle of relativity. Suppose that a physical law can not be expressed in terms of the space and time description, then by what criterion are we able to determine whether or not this law fulfills the principle of relativity? For instance, the law of conservation of energy-momentum is not expressed in terms of the space and time description. Yet, without a doubt, this law fulfills the principle of relativity.

According to quantum theory, a physical system is described by a state in Hilbert space subjected to certain laws. For instance, in quantum theory the spin of an elementary particle is unlike a spinning top in classical theory which has three well-defined components in the three dimensional physical world. Instead, the spin of an elementary particle is described as a state $|\mathbf{s}\rangle$ in Hilbert space obeying the spin rule $\hat{\mathbf{s}} \times \hat{\mathbf{s}} = i\,\hbar\,\hat{\mathbf{s}}$, where $\hbar = h/2\pi$, and h is Planck's constant. In addition, a system of identical particles is described by a state in a multi-dimensional configuration space obeying Pauli exclusion principle and quantum statistics of identical particles. Quantum statistics, the spin rule, Pauli exclusion principle and Heisenberg's uncertainty principle are not expressed as functions of space and time coordinates. Therefore, by the Lorentz covariance criterion, it is impossible to see whether or not these laws of quantum theory fulfill the principle of relativity. With the current interpretation of the principle of relativity, it is impossible to answer the question – Can quantum theory be consistent with the principle of relativity?

Are Planck's constant, Heisenberg's uncertainty principle, the spin rule, Pauli exclusion principle and quantum statistics the same in all inertial frames? Physical laws of a true theory must be the same in all inertial frames. Suppose quantum theory is indeed true for the description of physical phenomena, then laws of quantum theory must fulfill the principle of relativity. Consequently, the current notion of the principle of relativity must be radically rectified. A simple example is employed to illustrate that Heisenberg's uncertainty principle fulfills the principle of relativity. Consider a free particle moving with a definite momentum $\mathbf{p}$ and energy E with respect to a frame X. In quantum theory the motion of the particle is described as a state $|\mathbf{p}\rangle$ in Hilbert space. By Heisenberg's uncertainty principle $[\hat{x}_j, \hat{p}_k] = i\,\hbar\,\delta_{jk}$, the position observable $\hat{x}_j$ does not commute with the momentum observable $\hat{p}_j$. Thus, the exact position of the particle can not be exactly determined without disturbing the original state $|\mathbf{p}\rangle$. Suppose that this state is described in terms of an abstract wave function $\psi(\mathbf{r}, t) = \langle \mathbf{r}|\mathbf{p}\rangle$ with respect to the frame X. The position of the particle with respect to the frame X can only be predicted probabilistically as proportional to $|\psi(\mathbf{r}, t)|^2$. With respect to another frame X' uniformly moving relative to the frame X, the particle moves with a definite momentum $\mathbf{p}'$ and energy E', by relativistic transformation of energy-momentum, instead of space-time coordinates. With respect to the frame X', the particle is in the quantum state $|\mathbf{p}'\rangle$. By the same Heisenberg's uncertainty principle, the wave function of the particle is $\psi'(\mathbf{r}', t') = \langle \mathbf{r}'|\mathbf{p}'\rangle$ with respect to the frame X'. The wave function $\psi'(\mathbf{r}', t')$ of the particle with respect to the frame X' is not directly obtained from the wave function $\psi(\mathbf{r}, t)$ with respect to the frame X by the LT of space-time coordinates. Rather, the wave function of the particle is transformed via the transformation of energy-momentum. Thus, Heisenberg's uncertainty principle fulfills the principle of relativity, though its mathematical formula is not manifestly Lorentz-covariant.

Therefore, to fulfill the principle of relativity, physical laws are required to be the same in all inertial frames, rather than their mathematical formulas are Lorentz-covariant under the LT of space-time coordinates. To render quantum theory consistent with the principle of relativity, laws of quantum theory are *ab initio* hypothesized to be the same in all inertial frames, though their mathematical formulas are not manifestly Lorentz-covariant in the usual sense. Yet, the validity of this hypothesis depends on the results of experiments carried out on consequences derived from this

hypothesis.

IV. THE PRINCIPLE OF RELATIVITY AND THE LORENTZ COVARIANCE CRITERION

According to the principle of relativity, all inertial frames are physically equivalent, and thus physical laws are the same in all inertial frames [34–36]. However, there are different viewpoints on how physical laws should be formulated in order to fulfill the principle of relativity [37, 38]. The most accepted viewpoint is the Lorentz covariance criterion for the principle of relativity – to fulfill the principle of relativity, the mathematical formula of a physical law must be Lorentz covariant under the LT of space-time coordinates [36, 39, 40]. Nonetheless, as mentioned in the end of Sec. II, the Lorentz covariance criterion is not a sufficient condition for a physical law to fulfill the principle of relativity. In order to fulfill the principle of relativity, apart from the mathematical formula of a physical law is Lorentz covariant, all quantities in that equation must be defined and interpreted physically in the same way in all inertial frames. A physical law whose mathematical formula apparently satisfies the Lorentz covariance criterion does not guarantee that law fulfills the principle of relativity.

Furthermore, it was pointed out that the manifestly covariant equation $\partial_\alpha A^\alpha = 0$ does not imply A^α is a Lorentz-covariant 4-vector [41]. It is possible that the same equation $\partial_\alpha A^\alpha = 0$ holds in all inertial frames, but that equation is not Lorentz covariant subjected to the Lorentz covariance criterion, in the case that the quantity A^α in that equation is not covariant under the LT of space-time coordinates. Therefore, the Lorentz covariance criterion is not a necessary condition for a physical law to fulfill the principle of relativity. A physical law may fulfill the principle of relativity, though its mathematical formula does not satisfy the Lorentz covariance criterion.

To formulate relativistic physical theories, one usually substitutes the Lorentz covariance criterion for the principle of relativity [42–44]. However, a strict and universal application of the technique of covariance may result in mathematical formalisms that have no physical significance whatsoever [45]. Moreover, physical laws are rendered form-invariant via relativistic transformation of physical quantities, instead of space-time coordinates, in accordance with the new perspective on relativistic transformation. Contrary to the current viewpoint, the Lorentz covariance criterion is not essential to formulate relativistic physical theories.

V. CONCLUSION

The differential Lorentz transformation is the transformation of displacements $\mathrm{d}x^\alpha$ (equivalently, energy-momentum p^α) of a particle. Maxwell's equations of electrodynamic are rendered form-invariant through the transformation of electromagnetic fields in the k-space, rather than in the space-time space. The principle of relativity means that the same physical laws hold in all inertial frames, rather than their mathematical formulas are Lorentz-covariant under the Lorentz transformation of space-time coordinates. With the new perspective on relativistic transformation, physical laws need not to satisfy the Lorentz covariance criterion to fulfill the principle of relativity. The space and time concept underlying the new relativistic transformation is Newtonian absolute space and absolute time. With the new perspective on relativistic transformation and the very meaning of the principle of relativity, quantum theory becomes compatible with the principle of relativity. The compatibility between quantum theory and the principle of relativity as illustrated herein necessitates a radical reappraisal of the notion of space and time underlying Einstein's theory of special relativity.

Acknowledgments

We gratefully acknowledge Dr. C.M.L. Leonard for valuable comments during the preparation of this paper. Financial support from Taiwan National Science Council (Grant No. NSC100-2112-M-031-001) is acknowledged.

[1] D.Z. Albert and R. Galchen, Scientific American (March, 2010), p. 32.
[2] N. Gisin, Science **326**, 1357 (2009).
[3] N. Maxwell, Philosophy of Science. **52**, 23 (1985).
[4] D.J. Griffiths, *Introduction to Quantum Mechanics*, 2nd ed., (Pearson, Prentice Hall, 2005), sect. 3.5.3.
[5] E.V. Stefanovich, Found. Phys. **32**, 673 (2002).
[6] Young-Sea Huang, Physics Essays **5**, 159 (1992); Helv. Phys. Acta **66**, 346 (1993).

[7] M.P. Seevinck, *Can quantum theory and special relativity peacefully coexist?*, Invited white paper for quantum theory and the nature of reality, John Polkinghorne 80 Birthday Conference. St Annes College Oxford. 26-29 September, 2010.
[8] T. Maudlin, *Quantum non-locality and Relativity*, 2nd ed., (Blackwell Publishing, Malden, MA, 2008).
[9] M Sachs, Ann. Fond. L. de Broglie **31**, 383 (2006).
[10] W.C. Myrvold, Studies in History and Philosophy of Modern Physics **33**, 435 (2002).
[11] N. Gisin, arXiv:0512168v1[quant-ph] (2005).
[12] G.C. Ghirardi, arXiv:0912.0177v1[quant-ph] (2009).
[13] A. Shimony, *Controllable and Uncontrollable Non-Locality*, in *Proceedings of the International Symposium: Foundations of Quantum Mechanics in the light of New Technology*, S. Kamefuchi *et al* edited, (Physical Society of Japan, Tokyo, 1984), pp. 225-230.
[14] L.E. Ballentine and J.P. Jarrett, Am. J. Phys. **55**, 696 (1987).
[15] G. Nisticó and A. Sestito, Found. Phys. **41**, 1263 (2011).
[16] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935).
[17] Refer to the reference [16]. *Necessary condition for completeness*: Every element of the physical reality must have a counterpart in the physical theory. *Criterion for an element of reality*: If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.
[18] *Classical locality*: physical realities pertaining to a particle, and outcomes of subsequent measurements of the corresponding physical quantities, can not be altered by measurements carried out on another particle, provided these two particles are distantly separated far from the particles' interaction.
[19] N. Bohr, Phys. Rev. **48**, 696 (1935); Philosophy of Science **4**, 289 (1937).
[20] D. Bohm and Y. Aharonov, Phys. Rev. **108**, 1070 (1957).
[21] J. Bell, Physics **1**, 195 (1964); *Speakable and Unspeakable in Quantum Mechanics*, revised edition, (Cambridge University Press, Cambridge, 2004).
[22] J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[23] A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. **49**, 91 (1982).
[24] A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
[25] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1999).
[26] W. Tittel, J. Brendel, H. Zbinden and N. Gisin, Phys. Rev. Lett. **81**, 3563 (1999).
[27] D. Salart, A. Baas, C. Branciard, N. Gisin and H. Zbinden, Nature **454**, 861 (2008).
[28] Young-Sea Huang, Phys. Scr. **82**, 045011 (2010).
[29] For example, consider a particle moves on a *rough* surface. At any instant, the particle has an instantaneous velocity as specified by $\mathrm{d}x^\alpha$ which is not the differential of a non-existent function x^α. Another example, an infinitesimal heat $\mathrm{d}Q$ transferring to or from a system is not the differential of a non-existent heat function Q of the system.
[30] Young-Sea Huang, EPL **79**, 10006 (2007); Z. Naturforsch. **65a**, 615 (2010).
[31] Young-Sea Huang, Phys. Scr. **79**, 055001 (2009).
[32] Young-Sea Huang, Phys. Scr. **81**, 015004 (2010).
[33] Ø. Grøn and K. Vøyenli, Found. Phys. **29**, 1695 (1999).
[34] E. Giannetto, Hadronic J. Suppl. **10**, 365 (1995).
[35] C. Scribner, Jr., Am. J. Phys. **32**, 672 (1964).
[36] A. Einstein, *The Meaning of Relativity*, 5th ed., (Principle University Press, Princeton, 1974).
[37] V. Arunasalam, Physics Essays **10**, 528 (1997); **14**, 76 (2001); **21**, 9 (2008).
[38] T.E. Phipps, Jr., Physics Essays **21**, 16 (2008).
[39] R.P. Feynman, R.B. Leighton, and M.Sands, *The Feynman Lectures on Physics*, (Addison-Wesley, MA, 1963), Vol. I, chapter 15.
[40] J. Norton, Found. Phys. **19**, 1215 (1989).
[41] Young-Sea Huang, Can. J. Phys. **86**, 699 (2008).
[42] J.H. Field, Am. J. Phys. **69**, 569 (2001).
[43] P.M. Schwarz and J.H. Schwarz, *SPECIAL RELATIVITY: From Einstein to Strings*, (Cambridge University Press, Cambridge, 2004).
[44] E.G.P. Rowe, *Geometrical Physics in Minkowski Spacetime*, (Springer-Verlag, London, 2001).
[45] W. Yourgrau, Am. J. Phys. **33**, 984 (1965).